# UNM - PNM STATEWIDE MATHEMATICS CONTEST XLIII

November 5-8, 2010   First Round   Three Hours

1. For how many integers x in the set $\{1, 2, \ldots, 2010\}$ is $x^4 - x^3$ a cube?

2. In how many ways can 5 different prizes be awarded to 4 students so that each student receives at least one prize?

3. If $ABCD$ is a square with side AB of length 1cm and the segments AA_1, A_1B_1, C_1D_1 and D_1D all have length $\frac{1}{4}$cm, what is the area of the quadrilateral $EFGH$? See the figure for the location of the points.
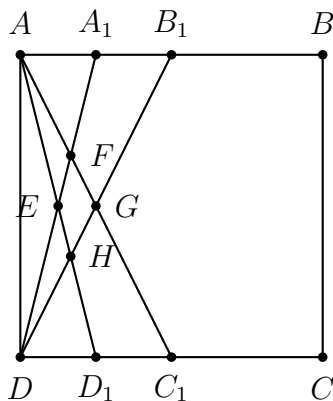
4. Let x and y be real numbers satisfying the equations

$$x + y = 1, \qquad (x^2 + y^2)(x^3 + y^3) = 26.$$

What is $x^2 + y^2$?

5. How many triples of real numbers (a, b, c) satisfy the equations

$$ab = c, \quad bc = a, \quad ca = b?$$

In other words, how many triples of real numbers (a, b, c) have the property that the product of any two equals the third of the numbers?

6. A palindrome is a number which reads the same backwards and forwards, for example 272 and 1331 are a palindromes. Diego decided to use a calculator to add the numbers from 1 to 15. Upon doing this he got a palindrome. Diego was certain that he entered every number correctly, except he forgot to add one of the numbers. What was the number he missed?

7. Semicircles are drawn on two sides of a rectangle $ABCD$ in which the longer side AB is twice the length of the shorter side AD, as shown. QAP is a line segment with segment QA of length $5cm$ and segment AP with length $24cm$. Compute the length of the shorter side.
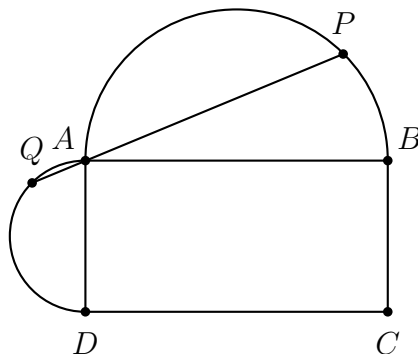
8. Find the radius of the largest disc that can be covered by three discs each of radius R.

9. Two cities A and B are 365 miles apart. A blue marker is placed every $36/365$ miles from A on the road to B. A red marker is placed every $14/365$ miles from A on the road to B. What is the shortest distance between markers of any color?

10. A school has a number of club activities that are practiced weekly during the school year which has thirty weeks. Before the start of the school year every student has to select at least one but no more than twelve different club activities. After the selection the student has to put the selected clubs in order which they follow cyclicly throughout the year attending each week the corresponding club from the chosen order. However the number of selected clubs of each student has to be such that the cycle is completed during the last week of classes. The following data was recorded during the school year for five chosen girls.

(i) The five chosen girls Bea, Donna, Ellen, Ginny and Izzy selected different number of clubs.

(ii) During the first week, Bea attended track, Donna and Ellen attended dance, Ginny attended mathematics and Izzy attended science.

(iii) During the eleventh week, two attended dance, one attended mathematics, one attended track and one attended history.

(iv) During the nineteenth week, Ginny attended mathematics and Izzy science, and the remaining girls went to dance.

(v) Ginny attended science during the twenty-second and history during the twenty-third week.

Who among these five girls attended the math club during the eleventh week?