

UNM - PNM STATEWIDE MATHEMATICS CONTEST LI

November 2-5, 2018 First Round Three Hours

1. Consider the degree 9 polynomial $(4x - 2)^9$. We can express this polynomial as $(4x - 2)^9 = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 + c_6x^6 + c_7x^7 + c_8x^8 + c_9x^9$. What is the sum $c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9$?
2. How many positive integer factors of 9,800,000 are not perfect squares?
3. List the number of two digit numbers $10 \leq x \leq 99$ satisfying the following properties: x is divisible by the sum of its digits and x divided by the sum of its digits gives 7.
4. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 0$ and

$$a_{n+1} = \frac{a_n - \sqrt{3}}{\sqrt{3}a_n + 1}$$

for $n = 1, 2, 3, \dots$. Calculate $a_1 + a_2 + a_3 + \dots + a_{2018}$.

5. Let E be the subset of the plane consisting of points (x, y) , where x, y are elements of the set $\{-1, 0, 1\}$. For example the point $(-1, 0)$ is in E . Let us pick any three points from E . Then what is the probability that there exist two points among these three points whose distance is $\sqrt{5}$?
6. King Hiero II of Syracuse wants to approximate the area of a circle but has forgotten the formula. His friend Archimedes suggests the following approximation to the area: take n equally spaced points $(P_1, P_2, \dots, P_{n-1}, P_n)$ on the circumference of the circle resulting in a n -sided regular polygon. Then, approximate the area of the circle by the area of this n -sided regular polygon. What is the approximation to the area of a circle with a diameter of 4 units if King Hiero II uses Archimedes method with $n = 8$ points?
7. There are two taps in a restroom which together fill a 100 gallon tank in 10 hours. However, Tap 1 takes 4 hours longer to fill a 50 gallon tank than it takes Tap 2 to fill a 30 gallon tank. What is the largest sized tank in gallons that can be filled by Tap 1 in 5 hours?
8. Janet decides to donate money to a charity every year. But being a mathematician, she decides that the number of dollars she donates to the charity would be equal to the total number of odd positive integers with distinct digits between 1 and the current year. (So, for example, 253 is one such number, but 799 is not since 9 is repeated).

How much money did Janet donate to the charity this year (that is, in 2018)?

9. Let A, B, C be three points on the plane. $A = (2, 1)$, the point B is on the x -axis, and the point C is on the line $x = y$. Suppose we can move B along the x -axis and C along the line $x = y$ freely. What is the minimum value of the perimeter of the triangle ABC ?
10. Let x_1, x_2, x_3 be nonnegative real numbers such that $x_1 + x_2 + x_3 = 1$. Calculate the maximum value of

$$(x_1 + 3x_2 + 5x_3)\left(x_1 + \frac{x_2}{3} + \frac{x_3}{5}\right).$$