

**UNM-PNM Statewide High School Mathematics Contest**  
**Round-1 Contest, 11-13 November 2023**

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- Please do the following **ten** problems over a continuous **three-hour period**.
- The solution to each problem is either an integer  $n$ , an expression  $n\pi$ , a fraction  $p/q$  in **lowest terms**, or an expression  $p\pi/q$ . As your answer, enter either  $n$  or the denominator  $q$ .

<b>Example problem</b>	<b>Solution</b>	<b>Entered answer</b>
What is the area of a square of side length 3?	9	9
What is the area of a square of side length $\frac{1}{2}$ ?	$\frac{1}{4}$	4
What is the circumference of a circle of radius 1?	$2\pi$	2
What is the circumference of a circle of radius $\frac{3}{7}$ ?	$\frac{6}{7}\pi$	7

- Because grading will be automated, all answers should be entered as integers. For example, write 2, not 2.0 or two. Do not include commas; write 100421, not 100,421.
  - Express angles in radians, not degrees.
  - No calculators permitted.
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**Student (LAST name, FIRST name):** \_\_\_\_\_

**Student email address:** \_\_\_\_\_

**Grade level:** \_\_\_\_\_

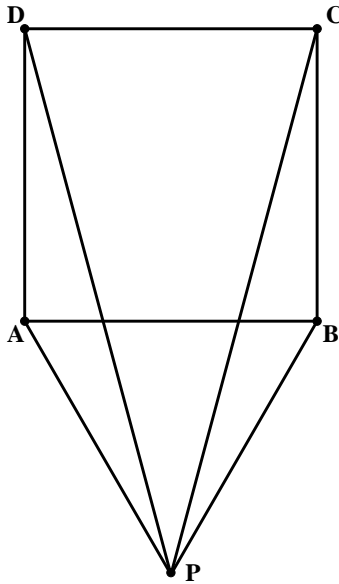
**School or club:** \_\_\_\_\_

**Teacher or coach:** \_\_\_\_\_

1. A hole is drilled in a sheet of aluminum. For 20 seconds the steel drill bit spins at 50 revolutions per second, and then for 10 seconds it spins at 80 revolutions per second until the hole is opened. For the purposes of the problem, you may view the transition from 50 to 80 revolutions per second as occurring instantaneously. What is the drill bit's average rate of rotation expressed in revolutions per second?

**ENTERED ANSWER:**

2. Suppose the point  $P$  outside of the square  $ABCD$  is such that  $\triangle ABP$  is an equilateral triangle. What is the angle  $\angle CPD$ ?



**ENTERED ANSWER:**

3. Find the largest value of  $x$  such that  $|x - 50| - |x - 112| \leq 0$ .

**ENTERED ANSWER:**

4. In double precision computer arithmetic, a real number occupies 64 bits or 8 bytes (a bit is either a 1 or a 0, and a byte is an 8-bit string). Suppose that a desktop computer has 8 GB ( $8 \times 10^9$  bytes) of random-access memory. What is the largest integer value  $n$  such that eight two-dimensional arrays, each of size  $n \times n$ , can fit in the computer memory? *Hint:*  $\sqrt{5} \simeq 2.23606797749979$ .

**ENTERED ANSWER:**

5. In the hexadecimal (base-16) number system a (nonstandard) notation for the base symbols is the following.

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, u, v, w, x, y, z$$

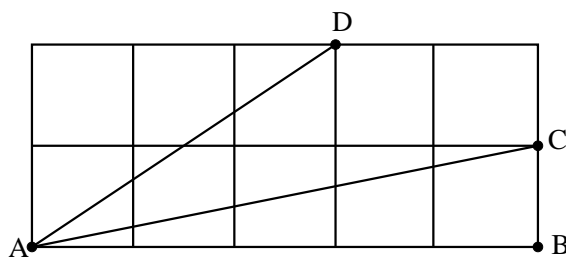
Find the decimal (base-10) representation of the following hexadecimal representation:  $(y420.0)_{16}$ .

**ENTERED ANSWER:**

6. Let a bowl have five marbles labeled 1, 2, 3, 4, 5. Suppose two marbles are drawn randomly without replacement from the bowl. Compute the probability that both marbles have an even number given that at least one of them does.

**ENTERED ANSWER:**

7. The rectangle shown is partitioned into 10 congruent squares, each  $1 \times 1$  in size. Find the sum of the two angles  $\angle BAC$  and  $\angle BAD$ .



**ENTERED ANSWER:**

8. How many integers between 1 and 2023 are multiples of 5, 7, or 17?

**ENTERED ANSWER:**

9. Let  $x_1, x_2, x_3$  denote the (possibly complex) roots of the cubic equation

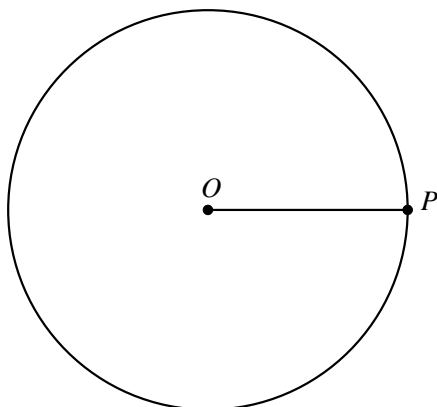
$$2x^3 + 12x^2 + 30x + 30 = 0.$$

Evaluate the the expression

$$x_1x_2 + x_1x_3 + x_2x_3.$$

**ENTERED ANSWER:**

10. Let  $C$  be a circle of radius 2 centered at the point  $O$ , with  $P$  a point on the circle. Define the region  $R$  to be all points  $Q$  inside the circle such that the triangle  $\triangle OPQ$  has an obtuse angle (one which exceeds  $\frac{1}{2}\pi$ ). What is the area of  $R$ ?



**ENTERED ANSWER:**