

UNM-PNM Solutions

- (a) How many distinct whole number factors does 6 have?
 - (b) How many distinct whole number factors does 12 have?
 - (c) How many distinct whole number factors does 144 have?
2. When there's nothing better to do, Vitali and JoJo meet up to wash Ms. Miriam's cat, Clementina. Vitali and JoJo noticed that after washing Clementina thirty-five times, the bar of soap's width shrunk by a factor of 2, the height shrunk by a factor of 3, and the length shrunk by a factor of 6. How many more times can JoJo and Vitali wash Clementina, before having to open up a new bar of soap?
3. In anticipation of the upcoming new year, four residents of San Jon sat down in the park and ate 2022 chiles. Each of the four people ate a different number of chiles. The two people who ate the second- and third- largest number of chiles ate 1011 chiles in total.
 - (a) How many chiles were eaten in total by the two people who ate the smallest and greatest number of chiles?
 - (b) What is the greatest possible number of chiles that could have been eaten by the San Jonian who ate the fewest chiles?
4. Inside unit square $ABCD$, quarter circle arcs with centers A and B are drawn. These arcs intersect at point X inside the square. If the distance from X to side AB is used as the side length of a square, what is its area?
5. Suppose $x, y, z > 0$ and $xyz = 1$. What is the difference between the maximum and minimum value the expression
$$\frac{1}{1+x+xy} + \frac{1}{1+y+yz} + \frac{1}{1+z+zx} \tag{1}$$
can attain?
6. A cube is inscribed in a sphere of radius 1. What is the surface area of the cube?
7. A natural number between 1 and 2021 (inclusive) is chosen with uniform probability. What is the probability that it is a perfect power, i.e. can be written as m^k , where m, k are natural numbers satisfying $m \geq 1, k > 1$?
8. If you were to write down the numbers $1, 2, \dots, 2021$ on a chalkboard, how many times would you write down the number 3?
9. Two people plan to meet at a particular location between 11:00 am and noon on a given day. They each arrive at a random time within the hour. They agree that if the first person there has to wait 15 minutes or more, that person will leave, and the two will not meet. What is the probability that they meet?
10. What is the product of the solutions to the equation $\left[\frac{2x-1}{3}\right] = \frac{x-1}{2}$? Here $[t]$ means the integer part of t , i.e., the greatest integer less than or equal to t . For example, $[-6.4] = -7$, $[6.4] = 6$, $[1] = 1$.